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NATURAL CONVECTION IN A CLOSED ROTATING CYLINDER

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NOMENCLATURE

k, thermal conductivity;
L, cylinder length;
Pr, Prandtl number;
Q, rate of heat transfer from surface;
Re, Reynolds number $(r_0\Omega)r_0/\nu$;
r, radial coordinate;
r_0, radius of cylinder;
T, temperature;
T_0, pure conduction solution;
T_1, T_2, boundary temperatures;
v_r, v_θ, v_z, velocity components;
z, axial coordinate.

Greek symbols

θ, angular coordinate;
ν, kinematic viscosity;
ρ, density;
ψ, stream function;
Ω, angular velocity.

INTRODUCTION

THE PROBLEM of rotationally induced laminar natural convection in a cylindrical container rotating about its own axis has evoked a number of analytical contributions, for example, [1–5]. These publications were motivated by applications such as turbine heat transfer, atmospheric re-entry, and rotating tubes and thermosyphons. In all of the aforementioned studies, analytical solutions were obtained by introducing appropriate simplifications into the mathematical model.

The present investigation was undertaken with the objective of treating the problem with a minimum of simplifying assumptions by solving numerically the full Navier–Stokes and energy equations. Although the project had to be terminated before the parametric objectives were fully realized, a number of interesting results were obtained and will be reported here.

A schematic diagram of the system to be analyzed is shown at the left of Fig. 1. All walls of the cylinder rotate with an angular velocity Ω. The temperatures of disks 1 and 2 are T_1 and T_2, respectively, and are uniform over the respective surfaces. The temperature of the cylindrical wall varies linearly between T_1 and T_2. Inasmuch as gravity effects will not be considered, the orientation of the cylinder is irrelevant as far as the analysis is concerned. For concreteness, $T_1 > T_2$.

In the absence of natural convection, the fluid within the cylinder would experience a pure rigid body rotation and heat conduction would be the sole mechanism of thermal transport. In the presence of natural convection, all three velocity components v_r, v_θ, and v_z are called into play, and heat conduction is supplemented by convective transport.

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ANALYSIS AND RESULTS

The starting point of the analysis was the five partial differential equations expressing conservation of mass, momentum, and energy in cylindrical coordinates (r, θ, z). To enable a natural convection motion, the Boussinesq model was introduced so that the density variation was accounted for in certain strategic terms, but otherwise all fluid properties were taken to be constant. In the present problem, the terms with variable density were $\rho v_\theta^2/r$ and $\rho v_r v_\theta/r$, which respectively appear among the inertia terms in the radial and tangential momentum equations.

The density–temperature relation used in evaluating the aforementioned terms was $\rho/\rho^* = T^*/T$, where ρ^* and T^* correspond to a reference state. This ρ, T relation is appropriate for a perfect gas under conditions where pressure variations are negligible, as was true for the parameter ranges of the analysis. In addition, in view of the small value of the Eckert number ($\ll 1$), the viscous dissipation and the compression work could properly be omitted from the energy equation.

With the specifications discussed in the foregoing paragraphs and with the assumption of axial symmetry ($\partial/\partial\theta = 0$), the governing partial differential equations were employed without further approximations. Numerical solutions were obtained by a finite-difference method that was an adaptation of that of [6], and full details of the method are presented therein.

As a prelude to the numerical solutions, it was necessary to specify the values of the following parameters: (a) the temperature ratio T_1/T_2, (b) the aspect ratio L/r_0, (c) the rotational Reynolds number $Re = (r_0\Omega)r_0/\nu$, (d) the Prandtl number Pr, and (e) the reference state for T^*. Although a wider range of parameter values was originally envisioned, the following cases were actually completed at the termination of the project: (a) $T_1/T_2 = 1{\cdot}05$; (b) $Re = 20$, 200, 1000 and 1500; (c) $L/r_0 = 0{\cdot}5$; (d) $Pr = 0{\cdot}7$, and (e) $T^* = (T_1 + T_2)/2$.

It is interesting to examine the range of Grashof numbers encompassed by the results. If the body force per unit mass is taken as the value of v_θ^2/r at $r = r_0$ and r_0 is selected as the characteristic dimension, then

$$Gr = Re^2 \frac{T_1 - T_2}{T^*}.$$

Thus, the Grashof number range extended from about 20 to 10^5.

A presentation of representative results will now be made. In Fig. 1, streamlines are plotted in the r, z plane for $Re = 200$ and 1500. These streamlines show the flow patterns for the radial and axial velocities v_r and v_z (that is,

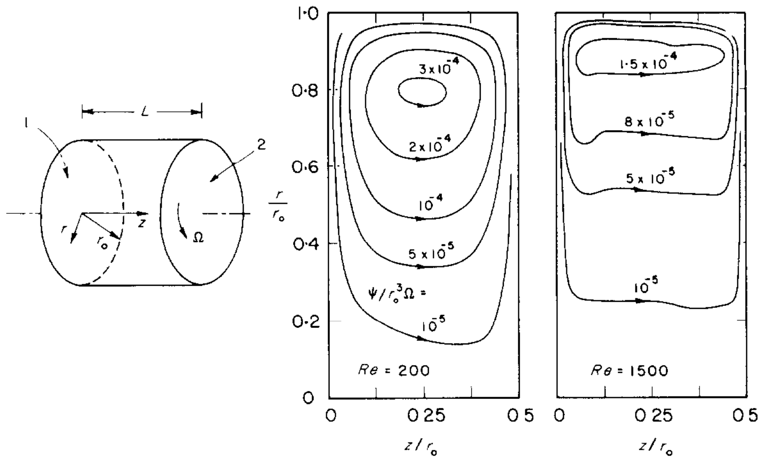

FIG. 1. Enclosure schematic (left diagram) and streamline patterns (center and right diagrams).

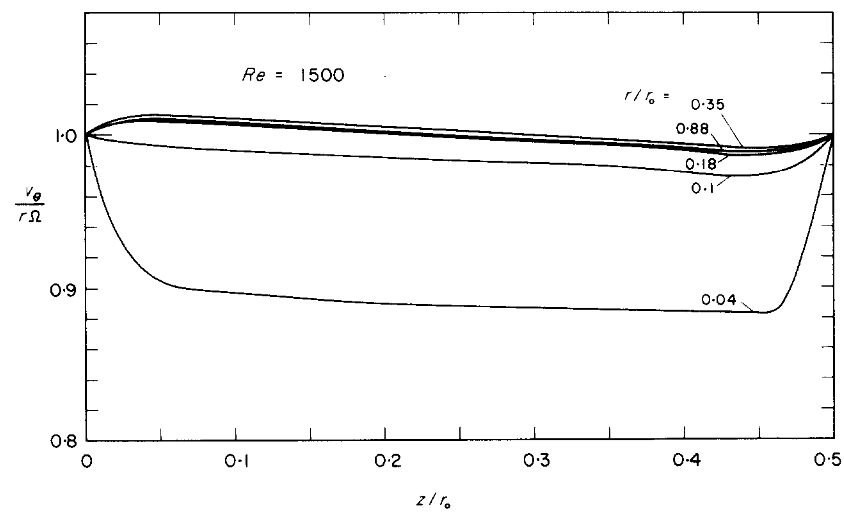

FIG. 2. Distributions of tangential velocity, $Re = 1500$.

$rv_r = -\partial\psi/\partial z$ and $rv_z = \partial\psi/\partial r$). The numerical values indicated on the curves are for $\psi/r_0^3\Omega$. The figure shows that natural convection gives rise to a cellular circulation pattern characterized by a radial outflow along the cooler disk 2 and a radial inflow along the hotter disk 1. In general, the radial velocities are primarily confined to the regions adjacent to the surfaces of the disk.

There are some marked differences in detail between the results for $Re = 200$ and those for $Re = 1500$. With increasing Reynolds number, the domains of radial and axial flow appear to have become sharply defined, with the former confined to very narrow layers near the disk surfaces and the latter holding sway over most of the gap between the disks. At lower Reynolds numbers, the domains are not quite as sharply delineated.

The effect of natural convection on the tangential velocity is illustrated in Fig. 2. The figure gives results for $Re = 1500$. The ordinate is the ratio of the local tangential velocity v_θ to the local rigid body velocity $r\Omega$ that would exist in the absence of natural convection. The abscissa is the dimensionless axial position variable z/r_0, and the curves are parameterized by the radial position variable. Inspection of the figure shows that significant deviations from rigid body rotation are confined to the region adjacent to the axis of the enclosure. For most of the span of the radial coordinate, the deviations are small and, in addition, are nearly independent of radial position.

Next, attention will be turned to the temperature distribution and surface heat transfer results. In the absence of natural convection, the temperature distribution is linear and is given by

$$T_0(z) = T_1 + (z/L)(T_2 - T_1)$$

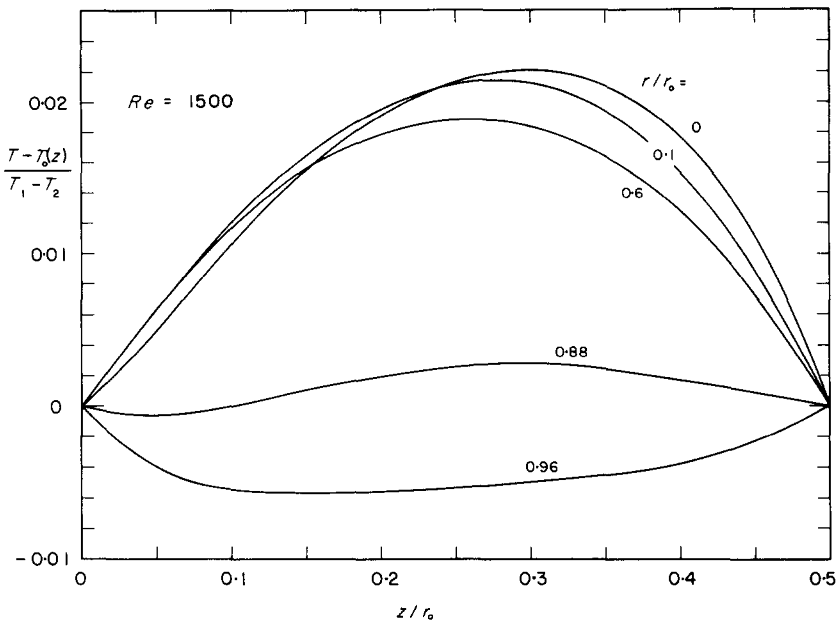

FIG. 3. Temperature deviations from pure conduction, $Re = 1500$.

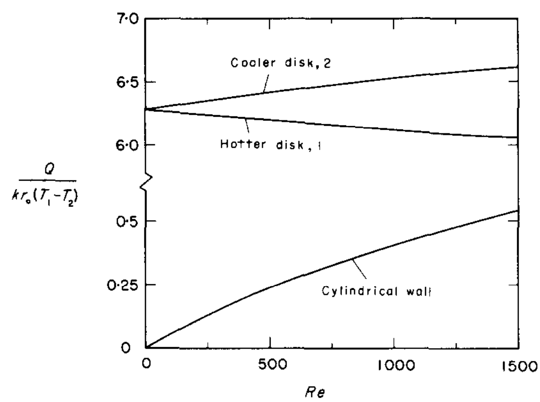

FIG. 4. Overall heat-transfer results.

where the subscript 0 denotes pure conduction. The local deviations from pure conduction may be expressed in dimensionless form by

$$\frac{T(r, z) - T_0(z)}{T_1 - T_2}$$

and representative distributions of this quantity are plotted in Fig. 3 for $Re = 1500$, with z/r_0 as abscissa and r/r_0 as curve parameter.

Study of these distributions reveals how the presence of the cellular flow patterns (Fig. 1) modifies the temperature field. From Fig. 1, it is seen that fluid flowing adjacent to the hotter disk subsequently penetrates into the core of the enclosure. Consequently, the temperatures in the core should exceed those for pure conduction, as is witnessed by the uppermost three curves in Fig. 3. It is further seen from Fig. 1 that fluid flowing along the colder disk is subsequently confined to a narrow region adjacent to the cylindrical wall. Correspondingly, T should be less than T_0 for a narrow range of radii near $r = r_0$. The lowermost curves of Fig. 3 corroborate this expectation.

The ordinate values of Fig. 3 are very much less than unity. This suggests that the actual temperature profiles, T vs z for parametric values of r, deviate very little from straight lines. Such profiles were plotted and carefully examined for the possible presence of thermal boundary layers on the disks, but none were in evidence.

Overall heat-transfer results for the two disks and the cylindrical wall are plotted in Fig. 4 as a function of the rotational Reynolds number. For the hotter disk 1 and the cylindrical wall, heat flows from the wall to the fluid, whereas for the cooler disk 2, the heat flow direction is from the fluid to the wall. The results indicate that the effect of natural convection is to increase the heat absorbed by the cooler disk while decreasing the heat loss from the hotter disk. These trends are consistent with the presence of the cellular flow as indicated in Fig. 1 and with the temperature deviations of Fig. 3. The heat loss from the cylindrical wall also increases as the natural convection circulation grows stronger.

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